

Saturation of Transverse Energy per Charged Hadron and Freeze-Out Criteria in Heavy-Ion Collisions

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Abstract. For beam energies from SPS to RHIC, the transverse energy per charged particle, E_T/N_{ch} , saturates at a value of approximately 0.8 GeV. A direct connection between this value and the freeze-out criterium $E/N \approx 1$ GeV for the primordial energy and particle number in the hadronic resonance gas model is established.

All relativistic heavy-ion experiments have so far confirmed the validity of $E/N \approx 1$ GeV as a freeze-out criterium, with E and N being, respectively the total energy and particle number of the primordial hadronic resonances before they decay into stable hadrons, i.e the energy E refers to the energy of all hadronic resonances like $\rho, \Delta, \omega, \dots$ and the number N refers to the total number of these particles at the chemical freeze-out point. These quantities can not be determined directly from experiment unless the final state multiplicity is low and hadronic resonances can be identified, which is not the case in relativistic heavy-ion collisions. It is thus not straightforward to link E/N to directly measurable quantities. In this paper we establish an approximate connection between E/N and the ratio of the pseudo-rapidity density of transverse energy and that of the charged particle yield, $(dE_T/d\eta/dN_{\text{ch}}/d\eta \equiv E_T/N_{\text{ch}})$, for beam energies ranging from about 1 AGeV up to 200 AGeV. In this energy range, E_T/N_{ch} at first increases rapidly from SIS [1] to AGS [2,3], then saturates to a value of about 800 MeV at SPS [4,5,6] energies and remains constant up to the highest available RHIC energies [7,8,9]. The present analysis of E_T/N_{ch} uses the hadron resonance gas model (thermal model) which describes the final state in relativistic heavy-ion collisions as composed of hadrons, including heavy hadronic resonances as being in thermal and chemical equilibrium. Our analysis therefore starts by relating the number of charged particles seen in the detector to the number of primordial hadronic resonances and the transverse energy to the energy E of primordial hadrons. The present status of E/N is shown in Fig. 1. The same results can also be plotted differently using energy density and baryon density as variables instead of T and μ_B . This brings out very clearly the maximum in the baryon density. At higher energies the baryon density goes to zero due to the vanishing of μ_B [13].

In this paper all thermal model calculations were performed using the THERMUS package [15]. At high energies the chemical freeze-out temperature saturates at a value of about 160 - 170 MeV as shown in Fig. 2 and at the same time the baryon chemical potential becomes very small [11]. As a consequence, several other quantities also become independent of beam energy. The average mass of hadronic resonances saturates at approximately the ρ mass at high energies as shown in Fig. 3. The ratio of all hadrons after resonance decays to the number

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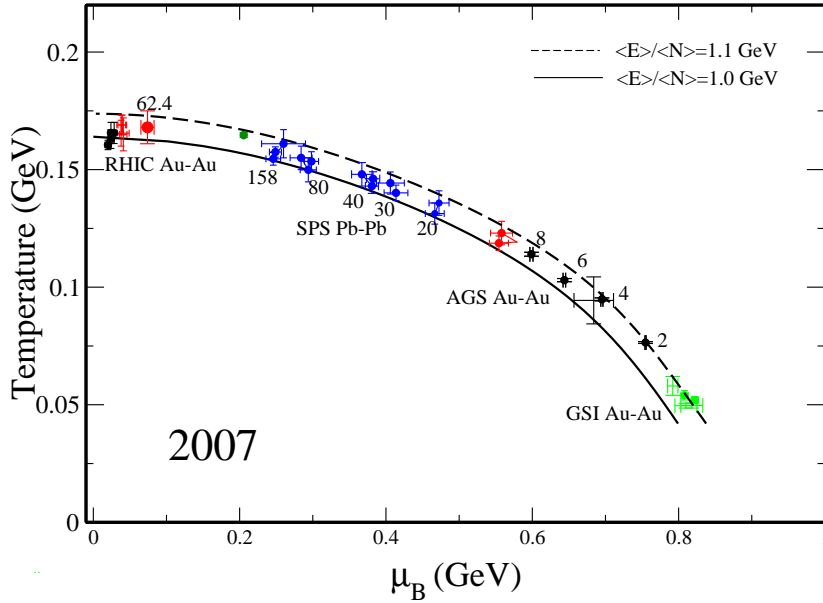


Fig. 1. Temperatures and baryon chemical potentials deduced from yields in relativistic heavy-ion collisions at various beam energies. The lower AGS points, denoted 2, 4, 6 and 8 still have to be confirmed. The solid lines correspond to constant values of E/N being kept fixed at 1 and 1.1 GeV respectively. The point at 62.4 GeV has been taken from Ref. [12].

of directly emitted hadrons at chemical freeze-out saturates at a value of about 1.7 as shown in Fig. 4. All of these are direct consequences of the saturation of the freeze-out temperature observed in Fig. 2 for increasing beam energies and the associated convergence of the baryon chemical potential to zero.

The transverse energy, $dE_T/d\eta$, is defined as the energy deposited transverse to the beam direction in a given interval of pseudo-rapidity η , since this quantity is integrated over usually, we will write E_T for brevity even though it has not been integrated over the full pseudo-rapidity interval. The transverse energy has two components, the hadronic one, E_T^{had} , and the electromagnetic one, E_T^{em} , coming from the electromagnetic particles (photons, electrons and positrons). Electromagnetic calorimeters are used to measure E_T^{em} whereas hadronic calorimeters or the Time Projection Chamber (for particle identification and momentum information) are used to measure E_T^{had} . The energy of a particle is defined as being the kinetic energy for nucleons, for anti-nucleons as the total energy plus the rest mass and for all other particles as the total energy [7,8,18].

In the experiments, the transverse energy and the charged particle multiplicity are measured in a similar way so that most of the systematic uncertainties cancel out in the ratio. Experiments have reported a constant value of the ratio $E_T/N_{\text{ch}} \sim 0.8$ GeV from SPS to RHIC [7,9], with the ratio being almost independent of centrality of the collision for all measurements at different energies. In all cases the value of E_T/N_{ch} has been taken for the most central collisions at mid-rapidity. At the end of this paper we consider the centrality dependence of E_T/N_{ch} . When this ratio is observed for the full range of center of mass energies, it shows two regions [9]. In the first region from lowest $\sqrt{s_{NN}}$ to SPS energy, there is a steep increase of the E_T/N_{ch} ratio with $\sqrt{s_{NN}}$. In this regime, the increase of $\sqrt{s_{NN}}$ causes an increase in the $\langle m_T \rangle$ of the produced particles. In the second region, SPS to higher energies, the E_T/N_{ch} ratio is very weakly dependent on $\sqrt{s_{NN}}$.

To estimate E_T/N_{ch} in the thermal model we relate the number of charged particles, N_{ch} , to the number, N , of primordial hadrons. To estimate the charged particle multiplicity at different center of mass energies from the thermal model, we proceed as follows. First we study

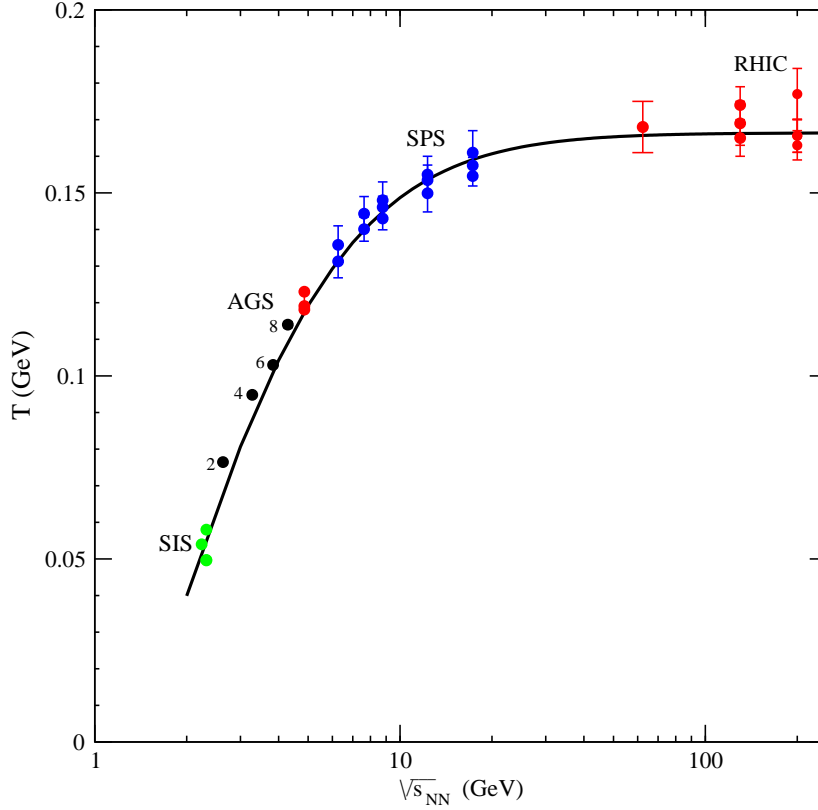


Fig. 2. Saturation of the chemical freeze-out temperature at high energies.

the variation of the ratio of the total particle multiplicity in the final state, N_{decays} , and that in the primordial i.e. N_{decays}/N with $\sqrt{s_{NN}}$. This ratio starts from one, since there are only very few resonances produced at low beam energy and becomes almost independent of energy after SPS energy. The value of N_{decays}/N in the region where it is independent of $\sqrt{s_{NN}}$ is around 1.7. The excitation function of N_{decays}/N is shown in Fig. 4(a). Secondly, we have studied the variation of the ratio of charge particle multiplicity and the particle multiplicity in the final state ($N_{\text{ch}}/N_{\text{decays}}$) with $\sqrt{s_{NN}}$. This is shown in Fig. 4(b). The $N_{\text{ch}}/N_{\text{decays}}$ ratio starts around 0.4 at lower $\sqrt{s_{NN}}$ and shows an energy independence at SPS and higher energies. At lower SIS energy, the baryon dominance at mid-rapidity makes $N_{\text{ch}}/N_{\text{decays}} \sim N_{\text{proton}}/N_{\text{(proton+neutron)}}$ which has a value of 0.45 for Au-Au collisions

As the next step we connect the transverse energy E_T to the the energy of the primordial hadrons E . In the hadronic resonance gas model there is a sum over all hadrons; furthermore, taking into account the experimental configuration which leads to adding the mass of the nucleon for anti-nucleons and subtracting the same for nucleons one has

$$\begin{aligned}
 \langle E_T \rangle \equiv & V \sum_{i=\text{Nucleons}} \int \frac{d^3p}{(2\pi)^3} (E_i - m_N) \sin \theta f(E_i) \\
 & + V \sum_{i=\text{Anti-nucleons}} \int \frac{d^3p}{(2\pi)^3} (E_i + m_N) \sin \theta f(E_i) \\
 & + V \sum_{i=\text{All Others}} \int \frac{d^3p}{(2\pi)^3} E_i \sin \theta f(E_i),
 \end{aligned}$$

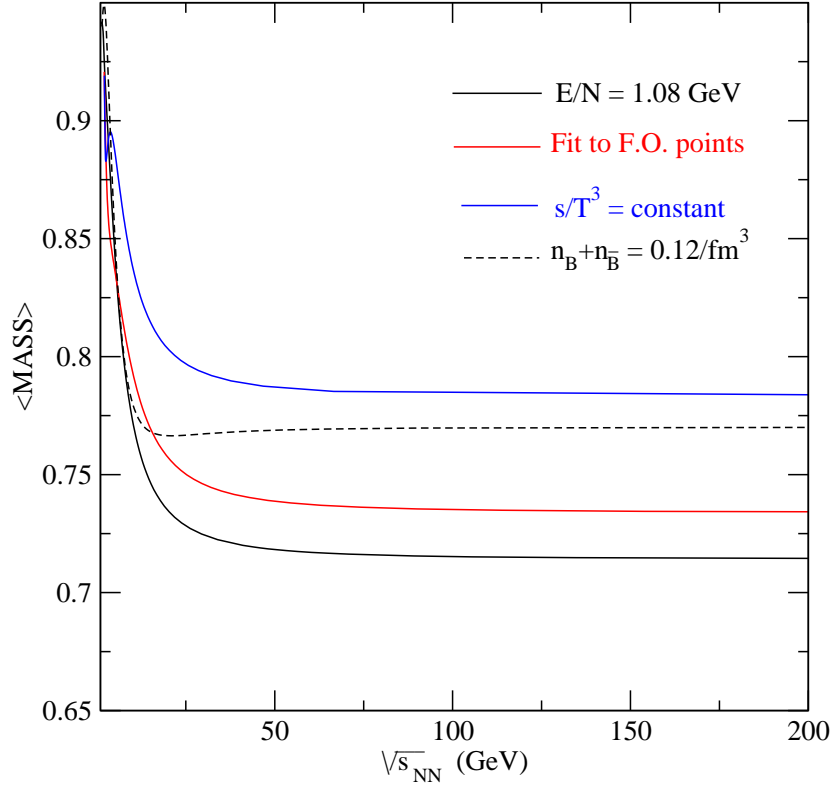


Fig. 3. Saturation of the average mass in the hadronic resonance gas model at high beam energies for various freeze-out criteria proposed in the literature [19,20,21,22].

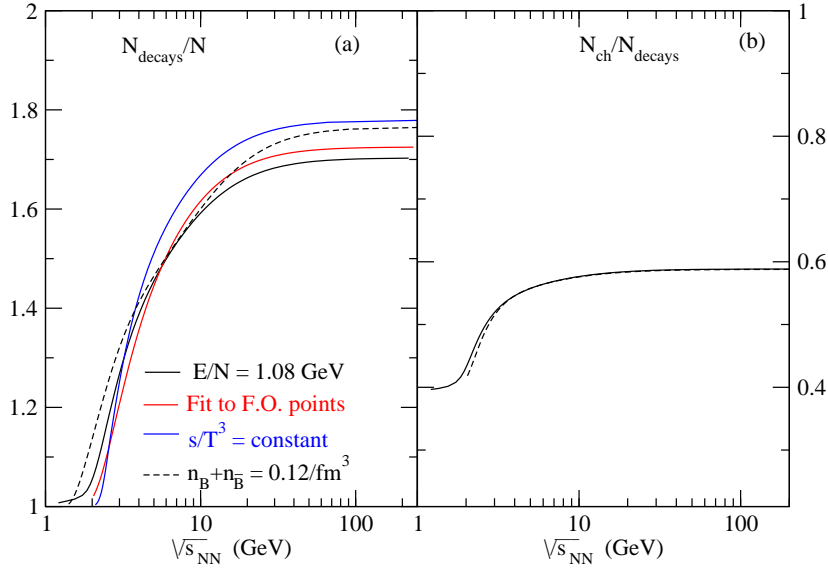


Fig. 4. Saturation of N_{decays}/N (a) and $N_{\text{ch}}/N_{\text{decays}}$ (b) with $\sqrt{s_{NN}}$. In (a) the results from various freeze-out criteria are indicated. In (b) the different freeze-out criteria give results that are indistinguishable.

$$= \frac{\pi}{4} [\langle E \rangle - m_N \langle N_B - N_{\bar{B}} \rangle]. \quad (1)$$

The above equation relates the transverse energy measured from the data and that estimated from the thermal model. In the limit of large beam energies one has

$$\begin{aligned} \lim_{\sqrt{s_{NN}} \rightarrow \infty} \frac{\langle E_T \rangle}{N_{\text{ch}}} &= \frac{\langle E_T \rangle}{0.6 N_{\text{decay}}}, \\ &= \frac{\pi}{4} \frac{1}{0.6} \frac{E}{1.7N}, \\ &= 0.77 \frac{E}{N}, \\ &\approx 0.83 \text{ GeV}. \end{aligned} \quad (2)$$

This value is close to the value measured at RHIC. It should be noted that the measured E_T will be affected by the transverse collective flow and by the difference between chemical freeze-out and kinetic freeze-out temperatures and therefore the description presented here is only a qualitative one. An analysis including flow was presented in Fig. 17 of the review article by Kolb and Heinz [16] who show that this improves the agreement with the data at SPS and RHIC beam energies. A detailed comparison in the framework of a specific model with a single freeze-out temperature, has been made in Ref. [17].

At higher energies, when μ_B nearly goes to zero, the transverse energy production is mainly due to the meson content in the matter. The intersection points of lines of constant E_T/N_{ch} and the freeze-out line give the values of E_T/N_{ch} at the chemical freeze-out. Hence at freeze-out, given the values of E_T/N_{ch} from the experimental measurements we can determine T and μ_B of the system.

For the most central collisions, the variation of E_T/N_{ch} with center of mass energy is shown in Fig. 5. The data have been taken from Ref. [1,2,3,4,5,6,7,8,9], and are compared with the corresponding calculation from the thermal model with chemical freeze-out. We have checked explicitly that other freeze-out criteria discussed in the literature give almost identical results for the behavior of E_T/N_{ch} as a function of $\sqrt{s_{NN}}$; this is the case for the fixed baryon plus anti-baryon density condition [19] and also for fixed normalized entropy density condition, $s/T^3 = 7$ [20,21,22]. We have checked explicitly that the centrality behavior is well reproduced by the thermal hadronic resonance gas model. [14].

In conclusion, we have discussed the connection between E_T/N_{ch} and the ratio of primordial energy to primordial particle multiplicity, E/N , from the thermal model. This model, when combined with chemical freeze-out criteria explains the data over all available measurements for the $\sqrt{s_{NN}}$ behavior of E_T/N_{ch} . It has to be noted that variables like E_T/N_{ch} , the chemical freeze-out temperature T_{ch} , $N_{\text{decays}}/N_{\text{primordial}}$ and $N_{\text{ch}}/N_{\text{decays}}$ discussed in this paper, show saturation starting at SPS and continuing to higher center of mass energies. This observation along with the centrality independence of E_T/N_{ch} is not inconsistent with the simultaneity of chemical and kinetic freeze-out at higher energies [24].

Acknowledgement

Three of us (JC, RS, DKS) would like to acknowledge the financial support of the South Africa-India Science and Technology agreement.

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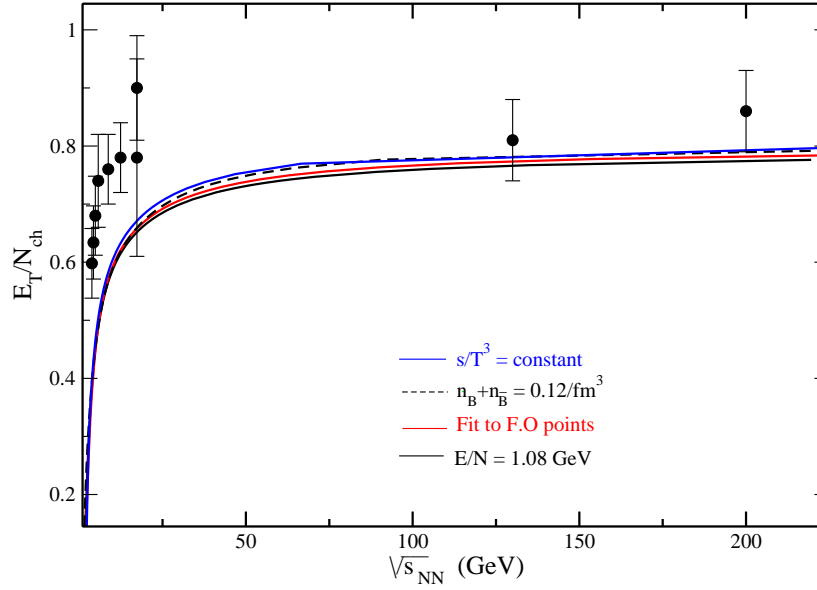


Fig. 5. Comparison between experimental data for E_T/N_{ch} with $\sqrt{s_{NN}}$ and the thermal model using $E/N = 1.08$ GeV as well as other freeze-out conditions [19,20,21,22].

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